

# Physics in Very Strong Magnetic Fields

## Introduction and Overview

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**Abstract** This paper provides an introduction to a number of astrophysics problems related to strong magnetic fields. The first part deals with issues related to atoms, condensed matter and high-energy processes in very strong magnetic fields, and how these issues influence various aspects of neutron star astrophysics. The second part deals with classical astrophysical effects of magnetic fields: Even relatively “weak” fields can play a strong role in various astrophysical problems, ranging from stars, accretion disks and outflows, to the formation and merger of compact objects.

**Keywords** magnetic fields · stars · accretion

### 1 Introduction

The subject “Physics in Very Strong Magnetic Fields” is a very broad one, and the title is also somewhat ambiguous. The first question one may ask is: *How strong a magnetic field is “Strong”?* The answer to this question will depend on the objects one is dealing with, on the issues one is interested in, and on whom one is talking to.

In the following, we will first review issues of strong magnetic fields from a general physics point of view and discuss how these issues may relate to some aspects of neutron star astrophysics. This focus on neutron stars reflects that fact that neutron stars are endowed with the strongest magnetic fields in the universe where fundamental strong-field physics can play an important role. It also reflects the author’s own research interest on the subject. For most other astrophysics problems, covering a wide range of sub-fields of

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astrophysics, magnetic fields are essentially classical, i.e., we are essentially dealing with Maxwell equations. We will discuss why such “weak” magnetic fields can be considered strong, and how such fields play an important role in various astrophysics contexts, ranging from stars and star formation, to disks and outflows, and to stellar mergers.

## 2 Atomic and Molecular Physics

When studying matter in magnetic fields, the natural (atomic) unit for the field strength,  $B_0$ , is set by equating the electron cyclotron energy  $\hbar\omega_{ce}$  to the characteristic atomic energy  $e^2/a_0 = 2 \times 13.6$  eV (where  $a_0$  is the Bohr radius), or equivalently by  $\hat{R} = a_0$ , where  $\hat{R} = (\hbar c/eB)^{1/2}$  is the cyclotron radius of the electron. Thus it is convenient to define a dimensionless magnetic field strength  $b$  via

$$b \equiv \frac{B}{B_0}; \quad B_0 = \frac{m_e^2 c^3}{\hbar^3} = 2.3505 \times 10^9 \text{ G}. \quad (1)$$

For  $b \gg 1$ , the cyclotron energy  $\hbar\omega_{ce}$  is much larger than the typical Coulomb energy, so that the properties of atoms, molecules and condensed matter are qualitatively changed by the magnetic field. In such a strong field regime, the usual perturbative treatment of the magnetic effects (e.g., Zeeman splitting of atomic energy levels) does not apply. Instead, the Coulomb forces act as a perturbation to the magnetic forces, and the electrons in an atom settle into the ground Landau level. Because of the extreme confinement ( $\hat{R} \ll a_0$ ) of the electrons in the transverse direction (perpendicular to the field), the Coulomb force becomes much more effective in binding the electrons along the magnetic field direction. The atom attains a cylindrical structure. Moreover, it is possible for these elongated atoms to form molecular chains by covalent bonding along the field direction. Interactions between the linear chains can then lead to the formation of three-dimensional condensates (see Lai 2001; Harding & Lai 2006 for review).

**(i) Atoms:** For  $b \gg 1$ , the H atom is elongated and squeezed, with the transverse size (perpendicular to  $\mathbf{B}$ )  $\sim \hat{R} = a_0/b^{1/2} \ll a_0$  and the longitudinal size  $\sim a_0/(\ln b)$ . Thus the ground-state binding energy  $|E| \simeq 0.16 (\ln b)^2$  (au) (where 1 au = 27.2 eV; the factor 0.16 is an approximate number based on numerical calculations). Thus  $|E| = 160, 540$  eV at  $B = 10^{12}, 10^{14}$  G respectively. In the ground state, the guiding center of the electron’s gyro-motion coincides with the proton. The excited states of the atom can be obtained by displacing the guiding center away from the proton; this corresponds to  $\hat{R} \rightarrow R_s = (2s+1)^{1/2} \hat{R}$  (where  $s = 0, 1, 2, \dots$ ). Thus  $E_s \simeq -0.16 \{\ln[b/(2s+1)]\}^2$  (au).

We can imagine constructing a multi-electron atom (with  $Z$  electrons) by placing electrons at the lowest available energy levels of a hydrogenic ion. The lowest levels to be filled are the tightly bound states with  $\nu = 0$  (zero node in the wavefunction along the field direction). When  $a_0/Z \gg \sqrt{2Z-1} \hat{R}$ , i.e.,  $b \gg 2Z^3$ , all electrons settle into the tightly bound levels with  $s = 0, 1, 2, \dots, Z-1$ .

Reliable values for the energy of a multi-electron atom for  $b \gg 1$  can be calculated using the Hartree-Fock method or density functional theory, which takes into account the electron-electron direct and exchange interactions in a self-consistent manner.

**(ii) Molecules and Chains:** In a strong magnetic field, the mechanism of forming molecules is quite different from the zero-field case. The spins of the electrons in the atoms are aligned anti-parallel to the magnetic field, and thus two atoms in their ground states do not bind together according to the exclusion principle. Instead, one H atom has to be excited to the  $s = 1$  state before combining (by covalent bond) with another atom in the  $s = 0$  state. Since the “activation energy” for exciting an electron in the H atom from  $s$  to  $(s + 1)$  is small, the resulting  $H_2$  molecule is stable. Moreover, in strong magnetic fields, stable  $H_3$ ,  $H_4$  etc. can be formed in the similar manner. The dissociation energy of the molecule is much greater than the  $B = 0$  value: e.g., for  $H_2$ , it is 40,350 eV at  $10^{12}$ ,  $10^{14}$  G respectively. A highly magnetized molecule exhibits excitation levels much different from a  $B = 0$  molecule.

**(iii) Neutron Star Atmospheres and Radiation:** An important area of research where the atomic physics in strong magnetic fields plays an important role is the study of neutron star (NS) atmospheres and their radiation (see Potekhin et al. 2014 for more details). Thermal, surface emission from isolated NSs can potentially provide invaluable information on the physical properties and evolution of NS (equation of state at super-nuclear densities, superfluidity, cooling history, magnetic field, surface composition, different NS populations). In recent years, considerable observational resources (e.g. *Chandra* and *XMM-Newton*) have been devoted to such study. For example, the spectra of a number of radio pulsars (e.g., PSR B1055-52, B0656+14, Geminga and Vela) have been observed to possess thermal components that can be attributed to emission from NS surfaces and/or heated polar caps. Phase-resolved spectroscopic observations are becoming possible, revealing the surface magnetic field geometry and emission radius of the pulsar. A number of compact sources in supernova remnants have been observed, with spectra consistent with thermal emission from NSs, and useful constraints on NS cooling physics have been obtained. Surface X-ray emission has also been detected from a number of SGRs and AXPs. Fits to the quiescent magnetar spectra with blackbody or with crude atmosphere models indicate that the thermal X-rays can be attributed to magnetar surface emission at temperatures of  $(3-7) \times 10^6$  K. One of the intriguing puzzles is the absence of spectral features (such as ion cyclotron line around 1 keV for typical magnetar field strengths) in the observed thermal spectra. Clearly, detailed observational and theoretical studies of surface emission can potentially reveal much about the physical conditions and the nature of magnetars.

Of particular interest are the seven isolated, radio-quiet NSs (so-called “dim isolated NSs”; see van Kerkwijk & Kaplan 2007; Haberl 2007). These NSs share the common property that their spectra appear to be entirely thermal, indicating that the emission arises directly from the NS atmospheres, uncontaminated by magnetospheric processes. Thus they offer the best hope for

inferring the precise values of the temperature, surface gravity, gravitational redshift and magnetic field strength. The true nature of these sources, however, is unclear at present: they could be young cooling NSs, or NSs kept hot by accretion from the ISM, or magnetars and their descendants. Given their interest, these isolated NSs have been intensively studied by deep *Chandra* and *XMM-Newton* observations. While the brightest of these, RX J1856.5-3754, has a featureless spectrum remarkably well described by a blackbody, absorption lines/features at  $E \simeq 0.2\text{--}2$  keV have been detected in six other sources. The identifications of these features, however, remain uncertain, with suggestions ranging from cyclotron lines to atomic transitions of H, He or mid-Z atoms in a strong magnetic field (see Ho & Lai 2004; Ho et al. 2008; Potekhin et al. 2014). Another puzzle concerns the optical emission: For four sources, optical counterparts have been identified, but the optical flux is larger (by a factor of 4-10) than the extrapolation from the black-body fit to the X-ray spectrum. Clearly, a proper understanding/interpretation of these objects requires detailed NS atmosphere modeling which includes careful treatments of atomic physics in strong magnetic fields.

### 3 Condensed Matter Physics

Several aspects of condensed matter physics in strong magnetic fields play an important role in neutron star astrophysics.

**(i) Cohesive Property of Condensed Matter:** Continuing our discussion of atoms/molecules in strong magnetic fields, as we add more atoms to a H molecular chain, the energy per atom in a  $H_n$  molecule saturates, becoming independent of  $n$ . We then have a 1D metal. Chain-chain interactions then lead to 3D condensed matter. The binding energy of magnetized condensed matter at zero pressure can be estimated using the uniform electron gas model. Balancing the electron kinetic (zero-point) energy and the Coulomb energy in a Wigner-Seitz cell (containing one nucleus and  $Z$  electrons), we find that the energy per unit cell is of order  $E \sim -Z^{9/5}b^{2/5}$ . The radius of the cell is  $R \sim Z^{1/5}b^{-2/5}$ , corresponding to the zero-pressure density  $\simeq 10^3 AZ^{3/5}B_{12}^{6/5}$  g cm $^{-3}$  (where  $A$  is the mass number of the ion).

Although the simple uniform electron gas model and its Thomas-Fermi type extensions give a reasonable estimate for the binding energy for the condensed state, they are not adequate for determining the cohesive property of the condensed matter. In principle, a three-dimensional electronic band structure calculation is needed to solve this problem. The binding energies of 1D chain for some elements have been obtained using Hartree-Fock method (Neuhauser et al. 1987; Lai et al. 1992). Density functional theory has also been used to calculate the structure of linear chains in strong magnetic fields (Jones 1986; Medin & Lai 2006a,b). Numerical calculations carried out so far indicate that for  $B_{12} = 1\text{--}10$ , linear chains are unbound for large atomic numbers  $Z \gtrsim 6$ . In particular, the Fe chain is unbound relative to the Fe atom; therefore, the chain-chain interaction must play a crucial role in determining

whether the 3D zero-pressure Fe condensed matter is bound or not. However, for a sufficiently large  $B$ , when  $a_0/Z \gg \sqrt{2Z+1}\hat{R}$ , or  $B_{12} \gg 100(Z/26)^3$ , we expect the Fe chain to be bound in a manner similar to the H chain or He chain (Medin & Lai 2006a,b). The cohesive property of magnetized condensed matter is important for understanding the physical condition of the polar gap and particle acceleration in pulsars (Medin & Lai 2007).

**(ii) Phase Diagram and Equation of State:** Given the energies of different bound states of a certain element, one can determine the phase diagram as a function of the field strength  $B$  and temperature. This is relevant to the outmost layer of neutron stars (NSs). For a given  $B$ , there is a critical temperature below which the phase separation will occur, and the NS surface may be in a condensed state, with negligible gas above it. Some isolated NSs with low surface temperatures may be in such a state (see van Adelsberg et al. 2005; Medin & Lai 2007).

Beyond zero-pressure density, the Coulomb interaction can be neglected, and the effects of magnetic field on the equation of state of matter depend on  $B$ ,  $\rho$  and  $T$ . We can define a critical “magnetic density”, below which only the ground Landau level is populated (at  $T = 0$ ), given by

$$\rho_B = 0.802 Y_e^{-1} b^{3/2} \text{ g cm}^{-3} = 7.04 \times 10^3 Y_e^{-1} B_{12}^{3/2} \text{ g cm}^{-3}, \quad (2)$$

where  $Y_e = Z/A$  is the number of electrons per baryon. We can also define critical “magnetic temperature”,

$$T_B \simeq \frac{\hbar\omega_{ce}}{k_B} \left( \frac{m_e}{m_e^*} \right) = 1.34 \times 10^8 B_{12} (1 + x_F^2)^{-1/2} \text{ K}, \quad (3)$$

where  $m_e^* = \sqrt{m_e^2 + (p_F/c)^2} = m_e \sqrt{1 + x_F^2}$ . There are three regimes characterizing the effects of Landau quantization on the thermodynamic properties of the electron gas:

(a)  $\rho \lesssim \rho_B$  and  $T \lesssim T_B$ : In this regime, the electrons populate mostly the ground Landau level, and the magnetic field modifies essentially all the properties of the gas. The field is sometimes termed “strongly quantizing”. For example, for degenerate, nonrelativistic electrons ( $\rho < \rho_B$  and  $T \ll T_F \ll m_e c^2/k_B$ , where  $T_F$  is the Fermi temperature), the pressure is  $P_e = (2/3)n_e E_F \propto B^{-2} \rho^3$ . This should be compared with the  $B = 0$  expression  $P_e \propto \rho^{5/3}$ . Note that for nondegenerate electrons ( $T \gg T_F$ ), the classical ideal gas equation of state,  $P_e = n_e k_B T$ , still holds in this “strongly quantizing” regime.

(b)  $\rho \gtrsim \rho_B$  and  $T \lesssim T_B$ : In this regime, the electrons are degenerate, and populate many Landau levels but the level spacing exceeds  $k_B T$ . The magnetic field is termed “weakly quantizing”. The bulk properties of the gas (e.g., pressure and chemical potential) are only slightly affected by such magnetic fields. However, the quantities determined by thermal electrons near the Fermi surface show large oscillatory features as a function of density or magnetic field strength. These de Haas - van Alphen type oscillations arise as successive Landau levels are occupied with increasing density (or decreasing magnetic field). With increasing  $T$ , the oscillations become weaker because of

the thermal broadening of the Landau levels; when  $T \gtrsim T_B$ , the oscillations are entirely smeared out, and the field-free results are recovered.

(c)  $T \gtrsim T_B$  or  $\rho \gg \rho_B$ : In this regime, many Landau levels are populated and the thermal widths of the Landau levels ( $\sim k_B T$ ) are higher than the level spacing. The magnetic field is termed “non-quantizing” and does not affect the thermodynamic properties of the gas.

**(iii) Transport Properties:** A strong magnetic field can significantly affect the transport properties and thermal structure of a neutron star crust. Even in the regime where the magnetic quantization effects are small ( $\rho \gg \rho_B$ ), the magnetic field can still greatly modify the transport coefficients (e.g., electric conductivity and heat conductivity). This occurs when the effective gyro-frequency of the electron,  $\omega_{ce}^* = eB/(m_e^* c)$ , where  $m_e^* = \sqrt{m_e^2 + (p_F/c)^2}$ , is much larger than the electron collision frequency  $1/\tau_0$ . When  $\omega_{ce}^* \tau_0 \gg 1$ , the electron heat conductivity perpendicular to the magnetic field,  $\kappa_\perp$ , is suppressed by a factor  $(\omega_{ce}^* \tau_0)^{-2}$ . In this classical regime, the heat conductivity along the field,  $\kappa_\parallel$ , is the same as the  $B = 0$  value. In a quantizing magnetic field, the conductivity exhibits oscillatory behavior of the de Haas - van Alphen type. On average, the longitudinal conductivity is enhanced relative to the  $B = 0$  value due to quantization. The most detailed calculations of the electron transport coefficients of magnetized neutron star envelopes are due to Potekhin (1999), where earlier references can be found (see Potekhin et al. 2014 for more details).

The thermal structure of a magnetized neutron star envelope has been studied by many authors (see Potekhin et al. 2014 for review). In general, a normal magnetic field reduces the thermal insulation as a result of the (on average) increased  $\kappa_\parallel$  due to Landau quantization of electron motion, while a tangential magnetic field (parallel to the stellar surface) increases the thermal insulation of the envelope because the Larmor rotation of the electron significantly reduces the transverse thermal conductivity  $\kappa_\perp$ . A consequence of the anisotropic heat transport is that for a given internal temperature of the neutron star, the surface temperature is nonuniform, with the magnetic poles hotter and the magnetic equator cooler (see, e.g., Shabaltas & Lai 2012 for a recent application).

#### 4 High Energy Physics: QED in Strong Magnetic Fields

In superstrong magnetic fields, a number of quantum-electrodynamics (QED) processes are important. A well-known one is single-photon pair production,  $\gamma \rightarrow e^+ + e^-$ . This process is forbidden at zero-field, but is allowed for  $B \neq 0$ , and is one of the dominant channels for pair cascade in pulsar magnetospheres (Ssurrock 1971; Medin & Lai 2010). Another process is photon splitting,  $\gamma \rightarrow \gamma + \gamma$ , which can attain appreciable probability for sufficiently strong fields. The critical QED field strength is set by  $\hbar\omega_{ce} = m_e c^2$ , i.e.,

$$B_Q = \frac{m_e^2 c^3}{e \hbar} = 4.414 \times 10^{13} \text{ G}. \quad (4)$$

Above  $B_Q$ , many of these QED effects become important.

A somewhat surprising strong-field QED effect is vacuum polarization, which makes even an “empty” space birefringent for photons propagating through it. This can significantly affect radiative transfer in neutron star atmospheres and the observed spectral and x-ray polarization signals even for modest field strengths. We discuss this issue below.

The magnetized plasma of a NS atmosphere is birefringent. A X-ray photon, with energy  $E \ll E_{ce} = \hbar\omega_{ce} = 1.16B_{14}$  MeV [where  $B_{14} = B/(10^{14} \text{ G})$ ], propagating in such a plasma can be in one of the two polarization modes: The ordinary mode (O-mode) has its electric field  $\mathbf{E}$  oriented along the  $\mathbf{B}$ - $\mathbf{k}$  plane ( $\mathbf{k}$  is along direction of propagation), while the extraordinary mode (X-mode) has its  $\mathbf{E}$  perpendicular to the  $\mathbf{B}$ - $\mathbf{k}$  plane. Since charge particles cannot move freely across the magnetic field, the X-mode photon opacity (e.g., due to free-free absorption or electron scattering) is suppressed compared to the zero-field value,  $\kappa_X \sim (E/E_{Be})^2 \kappa_{(B=0)}$ , while the O-mode opacity is largely unchanged,  $\kappa_O \sim \kappa_{(B=0)}$  (e.g., Meszaros 1992). Vacuum polarization can change this picture in an essential way. In the presence of a strong magnetic field, vacuum itself becomes birefringent due to virtual  $e^+e^-$  pairs. Thus in a magnetized NS atmosphere, both the plasma and vacuum polarization contribute to the dielectric tensor of the medium. The vacuum polarization contribution is of order  $10^{-4}(B/B_Q)^2 f(B)$  (where  $B_Q = m_e^2 c^3 / e \hbar = 4.414 \times 10^{13} \text{ G}$ , and  $f \sim 1$  is a slowly varying function of  $B$ ), and is quite small unless  $B \gg B_Q$ . However, even for “modest” field strengths, vacuum polarization can have a dramatic effect through a “vacuum resonance” phenomenon. This resonance arises when the effects of vacuum polarization and plasma on the polarization of the photon modes “compensate” each other. For a photon of energy  $E$  (in keV), the vacuum resonance occurs at the density  $\rho_V \simeq 0.964 Y_e^{-1} B_{14}^2 E^2 f^{-2} \text{ g cm}^{-3}$ , where  $Y_e$  is the electron fraction (Lai & Ho 2002). Note that  $\rho_V$  lies in the range of the typical densities of a NS atmosphere. For  $\rho \gtrsim \rho_V$  (where the plasma effect dominates the dielectric tensor) and  $\rho \lesssim \rho_V$  (where vacuum polarization dominates), the photon modes are almost linearly polarized — they are the usual O-mode and X-mode described above; at  $\rho = \rho_V$ , however, both modes become circularly polarized as a result of the “cancellation” of the plasma and vacuum polarization effects. When a photon propagates outward in the NS atmosphere, its polarization state will evolve adiabatically if the plasma density variation is sufficiently gentle. Thus the photon can convert from one mode into another as it traverses the vacuum resonance. The conversion probability  $P_{\text{conv}}$  depends mainly on  $E$  and atmosphere density gradient; for a typical atmosphere density scale height ( $\sim 1 \text{ cm}$ ), adiabatic mode conversion requires  $E \gtrsim 1\text{--}2 \text{ keV}$  (Lai & Ho 2003a). Because the O-mode and X-mode have vastly different opacities, the vacuum polarization-induced mode conversion can significantly affect radiative transfer in magnetar atmospheres. In particular, the effect tends to deplete the high-energy tail of the thermal spectrum (making it closer to blackbody) and reduce the width of the ion cyclotron line or other spectral lines (Ho & Lai 2003, 2004; Lai & Ho 2003a; van Adelsberg & Lai 2006). It is tempting to suggest that the absence of lines in the observed

thermal spectra of several AXPs is a consequence of the vacuum polarization effect at work in these systems.

We also note that even for “ordinary” NSs (with  $B \sim 10^{12}$ - $10^{13}$  G), vacuum resonance has a profound effect on the polarization signals of the surface emission; this may provide a direct probe of strong-field QED in the regime inaccessible at terrestrial laboratories (Lai & Ho 2003b; Wang & Lai 2009; see Lai 2010 for a review). Such polarization signals will be of interest for future X-ray polarimetry detectors/missions.

Finally, magnetic fields can modify neutrino processes that take place in neutron stars. For example, in proto-neutron stars with sufficiently strong B-fields, the neutrino cross sections and emission rates, as well as their angular dependences, can be affected, and these can contribute to the natal velocity kick imparted to the neutron star (e.g., Arras & Lai 1999a,b; Maruyama et al. 2014).

## 5 “Classical” Astrophysics

For most areas of astrophysics, magnetic fields are “classical”. That is, we are dealing with Maxwell’s equations, MHD and classical plasma physics. The quantization, microscopic effects discussed previous sections are not relevant. Nevertheless, these classical magnetic field effects are important, interesting and rich. We will highlight some of these in the following.

### 5.1 Clouds, Stars and Compact Objects

The first effect of “classical” magnetic fields is that they can influence the equilibrium of bound objects via the so-called magnetic Virial theorem. For a spherical cloud or star of mass  $M$  and mean radius  $R$ , static equilibrium requires that the ratio of the magnetic energy and gravitational energy be less than unity, i.e.,

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3 / 6}{GM^2 / R} \sim \frac{1}{6\pi^2 G} \left( \frac{\Phi}{M} \right)^2 \lesssim 1, \quad (5)$$

where the second equality assumes that the dominant internal magnetic field takes form of a large-scale poloidal field, and  $\Phi = \pi R^2 B_{\text{in}}$  is the magnetic flux threading the cloud.

In the context of star formation, clouds (cores) with  $E_{\text{mag}}/E_{\text{grav}} \gtrsim 1$  cannot collapse on a dynamical timescale, but require ambipolar diffusion to eliminate the magnetic flux. This process is perhaps relevant for the formation of low-mass stars (e.g., Shu et al. 1999), although in recent years the roles of turbulence in the molecular clouds have been recognized (McKee & Ostriker 2007).



For neutron stars (with  $M \simeq 1.4M_\odot$  and  $R \simeq 10$  km), equation (5) implies  $B_{\text{in}} \lesssim 10^{18}$  Gauss. This is the maximum field strength achievable in all astrophysical objects.

What do we know observationally about magnetic fields of isolated neutron stars? For radio pulsars, the dipole magnetic fields are inferred indirectly from the measured  $P$  and  $\dot{P}$  (rotation period and period derivative), and the assumption that the spindown is due to magnetic dipole radiation. For most “regular” pulsars, the magnetic fields thus derived lie in the range of  $10^{12-13}$  G. A smaller population, so-called “millisecond pulsars”, have fields in the range of  $10^{8-9}$  G. How such a “weak” field evolves from the regular field of  $10^{12-13}$  G remains unclear (see Payne & Melatos 2004). In recent years, a number of “High-B Radio Pulsars” have also been found: these have  $B \sim 10^{14}$  G, comparable to magnetars.

Magnetars are neutron stars powered by energy dissipation of magnetic fields. They usually have dipole fields (as inferred from  $P$  and  $\dot{P}$  based on x-ray timing) of  $B \gtrsim 10^{14}$  G. Interestingly, a number of low-field ( $\sim 10^{13}$  G) magnetars have also been found recently (Rea et al. 2010), although the internal fields could be higher. Indeed, there is growing evidence that there exist hidden magnetic fields inside neutron stars. This is the case for the neutron star in Kes 79 SNR: It has a dipole field of  $3 \times 10^{10}$  G, but the internal field buried inside its crust could be larger than  $10^{14}$  G, based on its observed large x-ray pulse fraction of 60% (Halpern & Gotthelf 2010; Shabaltas & Lai 2012; Viganò et al. 2013). In the case of SGR 0418+5729, the dipole field is less than a few times  $10^{12}$  G, but internal field could be much stronger (Turolla et al. 2011).

Another way to assess whether a magnetic field is “strong” is to look at the energetics. For magnetars, even in quiescence, the x-ray luminosity is  $L \sim 10^{34-36}$  erg s $^{-1}$ , much larger than the spindown luminosity ( $I\Omega\dot{\Omega}$ ). The giant flares of the three SGRs indicate that a much larger internal field is possible. For example, the December 2004 flare of SGR 1806-20 has a total energy of  $10^{46}$  erg, suggesting an internal field of at least a few times  $10^{14}$  G.

What is the origin of such strong magnetic fields? It is intriguing to note that (Reisenegger 2013) for upper main-sequence stars (radius  $10^{6.5}$  km), white dwarfs ( $10^4$  km) and neutron stars (10 km), the maximum observed magnetic fields ( $10^{4.5}$  G,  $10^9$  G and  $10^{15}$  G respectively) all correspond to similar maximum magnetic flux  $\Phi_{\text{max}} = \pi R^2 B_{\text{max}} \sim 10^{17.5-18}$  G km $^2$ . This seems to suggest a fossil origin of the strongest magnetic fields. However, recent observations of magnetic white dwarfs (and their populations in binaries) indicate the strong magnetic fields ( $\gtrsim$  a few MG) of white dwarfs originate from binary mergers (Wickramasinghe et al. 2014). So perhaps the strongest magnetic fields found in magnetars is the result of dynamo action in the proto-neutron star phase (Thompson & Duncan 1993). In any case, since  $E_{\text{mag}}/E_{\text{grav}} \lesssim 10^{-6}$  (assuming no significant hidden magnetic fields), these magnetic fields have a negligible effect on the global static equilibrium of the star.

## 5.2 Stellar Envelopes and “Outside”

Although astrophysically observed magnetic fields have a negligible effect on the global equilibrium of a star, they can strongly influence the local “static” equilibrium of stellar envelopes. A notable example is neutron star (NS) crust. Because of the evolution of crustal magnetic fields due to a combination of Hall drift and Ohmic diffusion, the NS crust can break (e.g. Pons & Perna 2011). This occurs when  $B^2/(8\pi) \gtrsim \mu\theta_{\max}$  (where  $\mu$  is the shear modulus and  $\theta_{\max}$  is the maximum strain of the crust), or  $B \gtrsim 2 \times 10^{14}(\theta_{\max}/10^{-3})^{1/2}$  G. The consequences of the crustal breaking (and its manifestations such as magnetar flares) are not clear. They depend on whether the breaking is fast or slow. The energy release and whether the energy can get out of the NS are also uncertain (see Link 2014; Beloborodov & Levin 2014).

Of course, outside the star, even a “weak” magnetic field can be quite “strong” and dominates the dynamics of the flow. Such magnetically dominated region is relevant to the magnetic braking of stars. In the case of radio pulsars, the electrodynamics and physical processes in the magnetosphere are ultimately responsible for most of the observed phenomena of pulsars. In recent years, there has been significant progress in *ab initio* calculations of pulsar magnetospheres (e.g. Tchekhovskoy et al. 2013), although it remains unclear whether the current theoretical approach can adequately explain some of the enigmatic pulsar phenomena (such as mode-switching in radiation; e.g. Hermsen et al. 2013). The magnetospheres of magnetars have also been studied: Unlike radio pulsars, the closed field line regions play an important role (e.g. Thompson et al. 2002; Beloborodov 2013).

Finally, further away from pulsars, we have pulsar wind nebulae, where pulsar wind impinges upon a supernova remnant, creating a broad spectrum of non-thermal radiation (from radio to gamma rays). The ultimate source of this radiation is the pulsar’s rotational energy, and magnetic field plays an important role in making such a “transfer of energy” possible (e.g. Amato 2013).

## 5.3 Accretion Disks

Magnetic fields play a number of important roles in accretion disks. First, we have magnetically dominated disks. These occur when  $B^2/(8\pi) \gtrsim \rho v_k^2/2$ , where  $v_k$  is the Keplerian velocity of the disk and  $\rho$  is the density. In the last few years, a number of studies have shown that the innermost region of a disk around a black hole may accumulate large magnetic flux, and relativistic jets can be generated through the Blandford-Znajek process (e.g. McKinney et al. 2012). However, a physical understanding of the state transition and jets (both steady and episodic) from black-hole x-ray binaries remains elusive (Fender & Belloni 2012; Yuan & Narayan 2014).

Outflows can be launched from disks with large-scale super-thermal magnetic fields (at the disk surface),  $B^2/(8\pi) \gtrsim \rho c_s^2/2$  (where  $c_s$  is the sound

speed). Such magnetocentrifugal winds/outflows (a la Blandford-Payne) may occur in x-ray binaries (in the thermal state) and in protostars (see Konigl & Pudritz 2000 for a review).

Such large-scale strong magnetic fields are unlikely to be produced in the disk by dynamo processes, and must be advected inward from large radii. This is an important issue that has received a lot of theoretical attention. The radial inward advection speed is  $|u_r| \sim \nu/r$ , and the outward Ohmic diffusion speed is  $u_{\text{diff}} \sim (\eta/H)(B_r/B_z)$ , where  $\nu$  is the disk viscosity,  $\eta$  is the magnetic diffusivity and  $H$  is the disk thickness. Clearly, the net outcome depends on the magnetic Prandtl number  $P_r = \nu/\eta$ , which is typically of order unity (based on local MRI turbulence simulations; Lesur & Longaretti 2009). Recent work has emphasized the importance of proper treatment of vertical structure of the disk (e.g., the electric conductivity is higher at the disk surface, so the field advection is faster than mass advection; Lovelace et al. 2009; Guilet & Ogilvie 2013).

Finally, even “weak” sub-thermal magnetic fields can play an important role in accretion disks. It is now well-established that for most astrophysical disks, MRI (magneto-rotational instability) driven turbulence is responsible for generating the anomalous viscosity needed for accretion to proceed (Balbus & Hawley 1998). It is also recognized that the strength of the turbulence depends on the net vertical field threading the disk (Hawley et al. 1995; Simon et al. 2013). Recent works have emphasized the roles of non-ideal MHD effects in suppressing turbulence in proto-planetary disks (Bai & Stone 2013; Bai 2014).

#### 5.4 Disk Accretion onto Magnetic Stars

Disk accretion onto magnetic central objects occurs in a variety of astrophysical contexts, ranging from classical T Tauri stars, and cataclysmic variables (intermediate polars), to accretion-powered X-ray pulsars. The basic picture of disk-magnetosphere interaction is well known: The stellar magnetic field disrupts the accretion flow at the magnetospheric boundary and funnels the plasma onto the polar caps of the star or ejects it to infinity. The magnetosphere boundary is located where the magnetic and plasma stresses balance,

$$r_m = \xi \left( \frac{\mu^4}{GM\dot{M}^2} \right)^{1/7}, \quad (6)$$

where  $M$  and  $\mu$  are the mass and magnetic moment of the central object,  $\dot{M}$  is the mass accretion rate and  $\xi$  is a dimensionless constant of order 0.5-1. Roughly speaking, the funnel flow occurs when  $r_m$  is less than the corotation radius  $r_c$  (where the disk rotates at the same rate as the star). For  $r_m \gtrsim r_c$ , centrifugal forces may lead to ejection of the accreting matter (“propeller” effect).

Over the years, numerous theoretical studies have been devoted to understanding the interaction between accretion disks and magnetized stars. Many

different models have been developed (see Lai 2014 for a review). In parallel to these theoretical studies, there have been many numerical simulations, with increasing sophistication. These simulations are playing an important role in elucidating the physics of magnetosphere-disk interaction in various astrophysical situations (see Romanova et al. 2014 and Zanni 2014 for review).

The problem of magnetosphere-disk interaction has many applications: (i) Rotation rate of protostars: Many protostars are found to have rotation rates about 10% of breakup. Magnetosphere spin equilibrium ( $r_m$  equals the corotation radius) has long been suggested, although magnetosphere/stellar winds may also play a role (Gallet & Bouvier 2013). (ii) Spinup/spindown of accreting x-ray pulsars: Many x-ray pulsars have been observed to exhibit changing spinup and spindown behavior over timescales of years. For example, 4U1626-67 is an accreting pulsar with spin period 7.66 s. The clean spinup before 1990.6 was followed by a clean spindown, and another spinup phase starting 2008.2. The spindown/spinup transition lasted 150 days. Understanding this spindown/spinup behavior and its correlation with the accretion rate remains an outstanding unsolved problem.

When the stellar field lines penetrate some region of the disk, they provide a linkage between the star and the disk. These field lines are twisted by differential rotation between the stellar rotation  $\Omega_s$  and the disk rotation  $\Omega(r)$ , generating a toroidal field. However, when the toroidal field becomes comparable to the poloidal field, the flux tube connecting the star and the disk will start expanding. This field inflation is driven by the pressure associated with the toroidal field. As the fields open up, the star-disk linkage is broken. Such field-opening behavior has been well-established through theoretical studies and numerical simulations in the contexts of solar flares and accretion disks (Lovelace et al. 1995). Given this constraint on the toroidal twist, steady-state disk-star linkage is possible only very near corotation. In general, we should expect a quasi-cyclic behavior, involving several stages: (1) The stellar field penetrates the inner region of the disk; (2) The linked field lines are twisted; (3) The resulting toroidal fields drive field inflation; (4) Reconnection of the inflated field restores the linkage. The whole cycle then repeats (see Aly & Juijpers 1990; Uzdensky et al. 2002). This quasi-cyclic behavior may be relevant to QPOs observed in low-mass X-ray binaries (see van der Klis 2006 for a review; Shirakawa & Lai 2002a,b) and other systems, as well as give rise to episodic outflows and winds (Zanni & Ferreira 2013).

Finally, we note that in the standard picture of magnetic star-disk interaction, it is usually assumed that the stellar spin axis is aligned with the disk axis (the disk normal vector). This seems reasonable since the star may have gained substantial angular momentum from the accreting gas in the disk. However, magnetic interaction between the star and the inner region of the disk may (if not always) change this simple picture (Lai 1999, 2003), giving rise to stellar spin - disk misalignment. This has application to spin-orbit misalignment in exoplanetary systems (Lai et al. 2011; Foucart & Lai 2011).

### 5.5 Magnetic Fields in the Formation of Compact Objects

In the “standard” picture of core-collapse supernovae leading to the formation of neutron stars, neutrino heating behind the stalled accretion shock, plus various hydrodynamical instabilities, are responsible for the explosion. Magnetic fields play a negligible role in this picture. However, there is a long list of theoretical works exploring the role of magnetic fields in supernovae (LeBlanc & Wilson 1970; Bisnovatyi-Kogan et al. 1976; Moiseenko et al. 2006; Burrows et al. 2007). The key requirement for the magnetic field to play a role is that the pre-SN core must have sufficiently rapid rotation – this is rather uncertain observationally. A technical challenge is that if one starts out with a modest magnetic field, and use MRI dynamo to amplify the field, it is important that the MRI scale is resolved in the numerical code – this is currently not achieved unless the initial field is greater than  $10^{15}$  G.

In general, newly formed magnetars can play two roles in supernovae. (i) They can power the explosion if the initial spin period of the proto-neutron star is less than  $\sim 3$  ms and the magnetic field is  $10^{15}$  G or higher (Bodenheimer & Ostriker 1974; Thompson et al. 2004). (ii) For modest rotation period ( $\sim 10$  ms), the released rotational energy does not affect the explosion itself, but can still impact the SN lightcurves (since the spindown timescale, about days to weeks, is comparable to the photon diffusion time through the remnant). Such energy injection may help explain some of the super-luminous SNe with  $L \gtrsim 10^{44}$  erg s $^{-1}$  (Kasen & Bildsten 2010; Woosley 2010). In this regard, it is of interest to note that many central compact objects in SNRs have been found to possess rather weak dipole fields ( $B \lesssim 10^{12}$  G) and slow rotation (period  $\sim 0.1$  s), although the internal fields may be much stronger.

Magnetic fields play an important role in the central engine of long Gamma-Ray Bursts (GRBs). Two scenarios are often discussed: (1) With rapid rotation, core collapse leads to the formation of a hyper-accreting black hole. Neutrino heating and magnetic fields (via Blandford-Znajek process) then lead to the production of relativistic jets (Zhang et al. 2003). (2) Core collapse leads to the formation of millisecond magnetars, which power the GRB outflows/jets. Recent observations of long-lasting ( $\sim 10^4$  s) x-ray emission/flares suggest that long-lasting central engine may be needed for some GRBs (Kumar & Zhang 2014). Also, the observed high polarization in reverse-shock emission indicates that large-scale magnetic fields are present in the GRB jets (Mundell et al. 2013).

### 5.6 Magnetic Fields in Merging Compact Binaries

There are two types of merging compact binaries that are of great interest: (1) NS/NS and NS/BH binaries: These produce gravitational waves that are detectable by LIGO/VIRGO and generate electromagnetic counterparts in the form of short GRBs and kilo-novae. (2) Compact WD/WD binaries: These produce various exotic outcomes (R CrB stars, AM CVn binaries, and possibly

accretion-induced collapse and SN Ia), and generate low-frequency gravitational waves detectable by LISA/NGO.

In recent years, there have been significant progress in simulating (in full General Relativity) the merger of NS/NS binaries (e.g. Shibata et al. 2006) and NS/BH binaries (Foucart et al. 2013). Simulations with magnetic fields are also becoming possible (Giacomazzo et al. 2011; Palenzuela et al. 2013), although much remains to be understood.

One issue of great interest is the merger of the NS magnetospheres prior to the merger of the stars. The combined binary system can behave as a single unipolar inductor producing radio waves that may be detectable (Hansen & Lyutikov 2001), although this is highly uncertain and detailed calculations are difficult. Nevertheless, a robust upper limit of the energy dissipation power in the magnetosphere that can be generated prior to NS merger can be obtained (Lai 2012). This upper limit indicates that the magnetospheric dissipation will not affect the orbital decay rate (and the gravitational waveform), although the prospect for radio detection remains uncertain.

Another issue of interest is the production of magnetic fields during NS/NS binary merger. Since the binary NSs cannot be spin-synchronized (because of the rapid orbital decay in the last few minutes of the binary lifetime), strong velocity shear is present when the two stars touch each other: Kelvin-Helmholtz instability develops at the interface, which may then lead to the generation of strong magnetic fields (Price & Rosswog 2006). Recent studies, however, suggest the dynamical impact of such magnetic fields may be limited to the shear layer (Obergaullinger, Aloy & Muller 2010), although the situation is not entirely clear (see Giacomazzo et al. 2014). Finally, the magnetic field in the merger remnant is of great importance. This situation is similar to the remnant in core-collapse supernova: Can the initial (weak/modest) magnetic field be amplified by differential rotation and MRI dynamo (Is MRI resolved in the simulation)? How are winds/outflows/jets produced? Is a black-hole or millisecond magnetar formed in the merger remnant?

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